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THOMSON REUTERS

[Maheswari* 4(6): June, 2017] ISSN 2349-4506
Impact Factor: 2.785

Global Journal of Engineering Science and Research Management

ESTIMATION OF RELIABILITY OF TIME DEPENDENT STRESS STRENGTH MODELS FOR NUMBER OF CYCLES

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DOI: 10.5281/zenodo.821336

KEYWORDS: Deterministic, random fixed, random independent, exponential distribution, stress- strength model, maximum likelyhood estimator.

ABSTRACT
In this paper, estimation of reliability of time dependent stress strength system for n number of cycles. The reliability is obtained when stress and strength follow case i) random independent stress and deterministic strength, case ii) Random independent stress and random independent strength, case iii) Random independent stress and random fixed strength. These models are useful for survival probability of number of cycles of heart diseases, cancer, dialysis of kidney diseases and etc.

INTRODUCTION
Estimation of system reliability has been discussed by Church, J.D., Harris, Bhattacharyya and Johnson and Ebrahimi..Stress strength model are studied by L.R.Lamberson and K.C.Kapoor. The time dependent models are studied by M.N.Gopalan etal .Most of these authors considered the strengths are independent and identically distributed random variables.. This paper is an attempt to obtain the estimation of system reliability when stress and strength follow exponential distribution for number of cycles. Time dependent stress strength models are considered with the repeated application of stress changing with time.

Let the random variables X and Y denote the stress and strength of the system. f(x) and g(y) are probability density functions of X and Y respectively . Then the reliability of the system at time t

$$R(t) = \int_0^\infty f(x)\left(\int_x^\infty g(y)dy\right)dx$$

There is uncertainty about the stress and strength random variables at any instant of time and also about the behavior of the random variables with respect to time and cycles. The three terms 'deterministic, 'random independent' and ‘random fixed’ are used to describe these uncertainties. Stress will vary randomly and will be independent from cycle to cycle only if it is being affected by other environmental factors. The failure of components under repeated stresses had been investigated primarily. Repeated stresses are characterized by the time, each load applied and the behavior of time intervals between the applications of loads. Individual expressions for reliability in different cases are derived.

STATISTICAL METHOD
The reliability of n number of cycles = R_n
Stress and strength follow exponential distribution, then p.d.f. of X& Y

$$f(x) = \lambda e^{-\lambda x},\ \ \lambda > 0\,, x > 0$$
$$g(y) = \mu e^{-\mu y},\qquad \mu > 0, y > 0$$

Case 1:
If strength is deterministic and stress is random independent

$$Then \qquad R_n = \left\{\int_0^{y_0} f(x)dx\right\}^n$$

$$R_n = \left(\int_0^{y_0} \lambda e^{-\lambda x}dx\right)^n = \left(1 - e^{-\lambda y_0}\right)^n$$

Case 2:

If strength is random independent and stress is random independent strength

$$Then \qquad R_n = \int_0^\infty f_i(x)\left(\int_x^\infty g_i\,(y)dy\right)dx$$

$$R_n = \int_0^\infty \lambda_i e^{-\lambda_i x}\left(\int_x^\infty \mu_i e^{-\mu_i y}\,dy\right)dx = \frac{\lambda_i}{(\lambda_i + \mu_i)}$$

Case 3:

If strength is random fixed and stress is random independent

$$Then\ \ R_n = \int_0^\infty g(y)\left(\int_0^y f(x)dx\right)^n \ dy$$

$$= \int_0^\infty \mu e^{-\mu y}\left(\int_0^y \lambda e^{-\lambda x}\right)^n \ dy$$

$$= \int_0^\infty \mu e^{-\mu y}\left(1 - e^{-\lambda y}\right)^n dy$$

$$R_n = \frac{n!\,\lambda^n}{(\mu+\lambda)(\mu+2\lambda)\ldots.(\mu+(n-1)\lambda)}$$

Estimation of parameter λ

The parameters λ is estimated by the method of maximum likelihood estimation (M.L.E)

$$L(\lambda, x_1, x_2, \ldots. x_n) = \prod_{i=1}^{n} \lambda \exp(-\lambda x_i) = \lambda^n \exp(-\lambda \sum_{i=1}^{n} x_i)$$

Then $\log L = n \log \lambda - \lambda \sum_{i=1}^{n} x_i$

For maximum L, $\quad \frac{\partial(logL)}{\partial\lambda} = 0$

$$\frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0$$

$$\therefore \hat{\lambda} = \frac{n}{\sum_{i=1}^{n} x_i}$$

The estimation of reliability for case 1:

$\widehat{R_i} = \left(1 - e^{-\hat{\lambda} y_0}\right)^n = \left(1 - e^{-\frac{n y_0}{\sum_{i=1}^{n} x_i}}\right)^n$

The estimation of reliability for case 2:

$$\hat{R}_i = \frac{\widehat{\lambda_i}}{(\lambda_i + \mu_i)}$$

The estimation of reliability for case 3:

$$\widehat{R_n} = \frac{n!\,\hat{\lambda}^n}{(\mu+\hat{\lambda})(\mu+2\hat{\lambda})\ldots.(\mu+(n-1)\hat{\lambda})}$$

$$The\ rth\ moment\ of\ X\ is \qquad E(X^r) = \int_0^\infty x^r f(x)dx$$

$$= \int_0^\infty x^r \lambda e^{-\lambda x} dx = \frac{\lambda\Gamma(r+1)}{\lambda^{r+1}} = \frac{\Gamma(r+1)}{\lambda^r}$$

Then

Mean= $E(X) = \frac{1}{\lambda}$, $E(X^2) = \frac{2}{\lambda^2}$ and $variance = V(x) = \frac{1}{\lambda^2}$

CONCLUSION

Estimation of reliability of time dependent stress strength system for n number of cycles has been obtained. The reliability is obtained when stress and strength follow exponential distribution for deterministic, random fixed and random independent stress.

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